Quantum Interference between a Single-Photon Fock State and a Coherent State

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Optical quantum computing employs beam splitters and Mach-Zehnder interferometers as basic building blocks [1], [2]. Normally Fock states generated by means of parametric down conversion are utilized. We generalize the input by combining a Fock state and a coherent state.

In an experiment in 2002 a displaced Fock state has been synthesized to a good approximation by overlapping a single-photon Fock state with a strong coherent pulse on a highly reflective beam splitter [3].

In the present work we consider the optical setups of Fig. 1, with a single-photon Fock state $|1\rangle$ and a coherent state $|\alpha\rangle$ incident on the input ports and derive exact analytical expressions for the quantum states at separate output ports. In the beam splitter case the corresponding output state and reduced density operator are

$$|1\rangle_0 |\alpha\rangle_1 \xrightarrow{BS} \hat{D}_2(r\alpha)\hat{D}_3(t\alpha)(t'\hat{a}_2^\dagger + r'\hat{a}_3^\dagger)|0\rangle_2 |0\rangle_3 \tag{1}$$

$$\hat{\rho}_3 = \mathrm{Tr}_2(\hat{\rho}_{23}) = |t'|^2 |t\alpha\rangle_3 \langle t\alpha|_3 + |r'|^2 \hat{D}_3(t\alpha)|1\rangle_3 \langle 1|_3 \hat{D}_3^\dagger(t\alpha). \tag{2}$$

The latter turns out to be a statistical mixture of a coherent state and a displaced Fock state. We also consider a Mach Zehnder Interferometer with the same input fields and show that we get an analogous analytical result at its output ports. The only differences are further dependencies on the parameters of a second beam splitter and an additional phase, located in path 3 and represented by the unitary phase shift operator $e^{i\theta\hat{n}_3}$. The reduced density operator in this case is

$$\hat{\rho}_4 = |\underline{v}|^2 |\underline{\beta}\rangle_4 \langle\underline{\beta}|_4 + |\underline{u}|^2 \hat{D}_4(\underline{\beta})|1\rangle_4 \langle 1|_4 \hat{D}_4^\dagger(\underline{\beta}), \tag{3}$$

where

$$\underline{v} := t_1' r_2 + e^{i\theta} r_1' t_2' \quad \underline{u} := t_1' t_2 + e^{i\theta} r_1' r_2' \quad \underline{\beta} := (r_1 t_2 + e^{i\theta} t_1 r_2')\alpha \tag{4}$$

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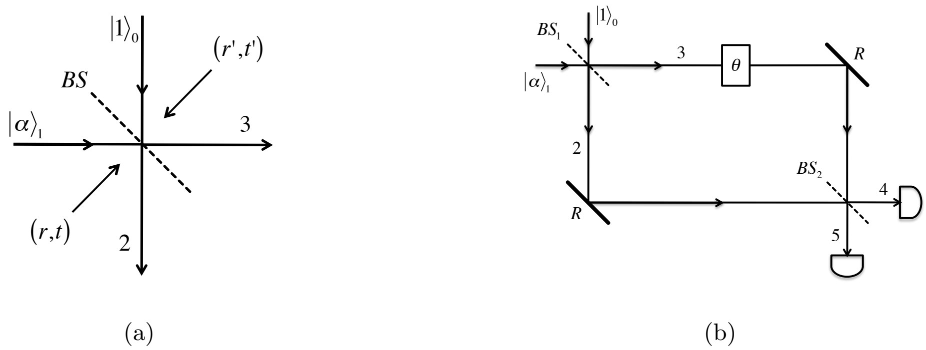

Fig. 1. (a) Beam Splitter and (b) Mach Zehnder interferometer with a phase shift in path 3. The input states in both cases are a single photon Fock state $|1\rangle_0$ and a coherent state $|\alpha\rangle_1$.

We realize that Eq. (3) is equivalent to Eq. (2). Therefore, as in the case of the beam splitter, the result is a mixed state between a coherent state and a displaced Fock state. The Wigner function of these states can be calculated easily.

For the mean photon number in port 4 we get

$$\langle n \rangle_4 = |\underline{u}|^2 + |\underline{\beta}|^2 . \tag{5}$$

An analogous result is obtained for port 5.

Here we want to mention that the calculations can be generalized for a Mach Zehnder Interferometer with a coherent state at one input port and an arbitrary state $|\psi\rangle$ at the other. As in the case of a single beam splitter, the output state is merely displaced compared to a Mach Zehnder with vacuum input instead of the coherent input state. This has been known to be true only in the limiting case of a highly reflectively beam splitter [3]. We show that this result is generally true. Potentially it might find application in future optic computing architectures.

References

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