Simple Sets of Measurements for Universal Quantum Computation and Graph State Preparation

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Abstract. We consider the problem of minimizing resources required for universal quantum computation using only projective measurements. We show that the set of observables $\{Z \otimes X, (\cos\theta)X + (\sin\theta)Y \text{ all } \theta \in [0, 2\pi)\}$ with one ancillary qubit is universal for quantum computation. The set is simpler than a previous one in the sense that one-qubit projective measurements described by the observables in the set are ones only in the (X, Y) plane of the Bloch sphere. The proof of the universality immediately implies a simple set of observables that is approximately universal for quantum computation. Moreover, the proof implies a simple set of observables for preparing graph states efficiently.

1 Introduction and Summary of Results

In 2001, Raussendorf and Briegel proposed a new model for quantum computation, which is called cluster state computation. Later, in 2003, Nielsen proposed a new model, which is called teleportation-based quantum computation. In contrast to conventional models, such as the quantum circuit model, these new models use only projective measurements for universal quantum computation and thus suggest a new way of realizing a quantum computer. Minimizing the resources required for universal quantum computation is important for realizing a quantum computer based on these new models.

We consider the problem under the assumption that we can use only projective measurements and do not have initial cluster states [1, 2]. The resources we focus on are observables, which describe projective measurements, and ancillary qubits. In 2005, Jorrand and Perdrix showed that the set of observables $\{Z \otimes X, Z, (\cos\theta)X + (\sin\theta)Y \text{ all } \theta \in [0, 2\pi)\}$ with one ancillary qubit is universal for quantum computation [1]. It has not been known whether a simpler universal set of observables can be constructed without increasing the number of ancillary qubits.

We show that the set of observables $\mathcal{S}_1 = \{Z \otimes X, (\cos\theta)X + (\sin\theta)Y \text{ all } \theta \in [0, 2\pi)\}$ with one ancillary qubit is universal. The set is simpler than Jorrand and Perdrix's [1] in the sense that one-qubit projective measurements described by the observables in $\mathcal{S}_1$ are ones only in the (X, Y) plane of the Bloch sphere. In the proof of the universality, the key idea is to use Y-measurements appropriately in place of other one-qubit projective measurements. In contrast to Jorrand and Perdrix's proof [1], our proof connects a simple universal set of observables with a simple approximate universal one. More precisely, our proof immediately implies the best known result for the approximate universality by Perdrix [2] that a set of two one-qubit observables and one two-qubit observable with one ancillary qubit is approximately universal. Such an example is the set of observables $\mathcal{S}_2 = \{Z \otimes X, Y, (X+Y)/\sqrt{2}\}$.

We also consider the problem of minimizing the resources required for preparing graph states efficiently. It is important to investigate this problem since graph states play a key role in quantum information processing. Høyer et al. showed that, for any graph $G = (V, E)$, some signed graph state $|G\rangle$ can be prepared by a quantum circuit consisting of one-qubit and two-qubit projective measurements with size $O(|V| + |E|)$, depth $O(|E|)$, and one ancillary qubit [3]. Even if the circuit uses the set of observables in [2], two one-qubit observables and one two-qubit observable are required.

Using the proof of the universality of $\mathcal{S}_1$, we show that the set of observables $\mathcal{S}_3 = \{Z \otimes X, Y\}$ with one ancillary qubit is sufficient for preparing graph states efficiently. More precisely, for any graph $G = (V, E)$, the (exact) graph state $|G\rangle$ can be prepared by a quantum circuit consisting of one-qubit and two-qubit projective measurements described by the observables in $\mathcal{S}_3$ with size and depth $O(|V| + |E|)$ and one ancillary qubit. The depth is $O(|E|)$ for the graphs in which we are interested. Though the usual method for preparing graph states performs controlled-Z operations, it is difficult to do so since $\mathcal{S}_3$ has only $Z \otimes X$ and Y. The key idea is to perform operations similar to controlled-Z operations and to remove the side effects of the similar operations by using Y-measurements.

References

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