Simulating sparse Hamiltonians with star decompositions*

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Abstract

Quantum simulation of Hamiltonian dynamics is a well-studied problem [1–3] and is one of the main motivations for building a quantum computer. Since the best known classical algorithms for simulating quantum systems are inefficient, this was the original application of quantum computers [4]. Besides simulating physics, Hamiltonian simulation has many algorithmic applications, such as adiabatic optimization, unstructured search, and the implementation of quantum walks.

The input to the Hamiltonian simulation problem is a Hamiltonian H and a time t; the problem is to implement the unitary operator e^{-iHt}. We say that a Hamiltonian acting on an N-dimensional quantum system can be simulated efficiently if there is a quantum circuit using $\mathrm{poly}(\log N, t, 1/\epsilon)$ one- and two-qubit gates that approximates (with error at most ϵ) the evolution according to H for time t.

Lloyd presented a method for simulating quantum systems that can be described by a sum of local Hamiltonians [1]. A Hamiltonian is called local if it acts non-trivially on at most a fixed number of qubits, independent of the size of the system.

This was later generalized by Aharonov and Ta-Shma [2] to the case of sparse (and efficiently row-computable) Hamiltonians. A Hamiltonian is sparse if it has at most $\mathrm{poly}(\log N)$ nonzero entries in any row. It is efficiently row-computable if there is an efficient procedure to determine the location and matrix elements of the nonzero entries in each row.

The complexity of this simulation was improved by Childs [5] and further improved by Berry, Ahokas, Cleve and Sanders [3]. Their algorithm has query complexity $(d^4(\log^* N)\,\|H\|)^{1+o(1)}$, where d is the maximum degree of the graph of the Hamiltonian H. These algorithms decompose

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the Hamiltonian into a sum of Hamiltonians, each of which is easy to simulate.

We present a different method of decomposing the Hamiltonian. We decompose a general sparse Hamiltonian into a small sum of Hamiltonians, each of whose graph of non-zero entries is a forest of star graphs. This is done using ideas from distributed computing for decomposing a graph into a sum of forests [8] and vertex coloring forests [9, 10]. We then show that a Hamiltonian whose graph of non-zero entries is a forest of stars can be efficiently simulated. This leads to an algorithm with query complexity $(d^2(d + \log^* N) \|H\|)^{1+o(1)}$.

The simulation of Ref. [3] has also been improved using a completely different approach [6, 7]. That algorithm is more efficient in terms of all parameters except the error ϵ, on which its dependence is considerably worse. The algorithm we present here maintains the same dependence on ϵ as in Ref. [3], providing the best-known method for high-precision simulation of sparse Hamiltonians.

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