Optimal pure state estimation as a randomized universal cloning

Giulio Chiribella[1]

Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario, Canada N2L 2Y5.

The connection between quantum estimation and cloning is an inspiring leitmotiv of Quantum Information Theory [1–8]. The main related question is: how well can we simulate cloning via estimation? Or, more precisely, how well can we simulate cloning with a "measure-and-prepare" (M&P) protocol where the input systems are measured, and the output systems are prepared in some state depending on the measurement outcome? As a particular instance of this question, one can ask whether "asymptotic cloning is state estimation" [9], that is, whether the gap between the single-particle fidelity of an optimal cloning channel and the fidelity of the corresponding optimal estimation vanishes when the number of clones tends to infinity.

In Ref. [7] Bae and Acín showed that a channel producing an infinite number of indistinguishable clones must be of the "measure-and-prepare" form. On the other hand, Ref. [8] proved that a channel producing $M < \infty$ indistinguishable clones can be simulated by an M&P channel introducing an error at most of order $\mathcal{O}(1/M)$ on each clone. The proof of Ref. [8] was based on the so-called *finite quantum de Finetti theorem* [10–12], that states that the restriction to k-particles of a permutationally invariant M-partite state can be approximated with an error at most of order $\mathcal{O}(k/M)$ by a mixture of product states of the form $\rho^{\otimes k}$.

Apparently, de Finetti theorems are the key to prove the equivalence between cloning and estimation. In this talk I will explore the converse path, showing a particular relation between the optimal estimation of an unknown pure state [13, 14] and the optimal universal cloning [2] that allows for an alternative derivation of the standard finite de Finetti theorem and also gives an hint to derive new de Finetti theorems. To this purpose I will consider a particular case of the following primitive:

Given two sets of pure states $\{\varphi_x\} \subseteq \mathcal{H}_{in}$ and $\{\psi_x\} \subseteq \mathcal{H}_{out}$,
transform the state φ_x into the state ψ_x with an M&P protocol.

Precisely, I will analyze in detail the case where the input set consists of M copies of an unknown pure state in dimension d, whereas the output set consists in $k \leq M$ copies of the same state. In this setting it is easy to show that the optimal M&P protocol is achieved by the optimal state estimation followed by the preparation of k copies of the estimated state. The key result here is an explicit representation of the optimal M&P channel as a random loss of $M - s$ particles followed by universal cloning from s to k copies. This representation will be used to provide an alternative proof of the standard finite quantum de Finetti theorem, and to derive norm bounds on the asymptotic convergence of quantum cloning to state estimation. Moreover, I will consider channels that distribute quantum information to M indistinguishable users, and show that the restriction to k users of any such channel has a quantum capacity that vanishes at rate $\mathcal{O}(k/M)$ when the ratio k/M tends to zero.

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